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STEADY HEAT TRANSFER TO A THIN INFINITE DISK WITH A CUT-OUT OPENING

Yu. I. Babenko

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A method is proposed which allows one to find the steady temperature gradient at the rim of a round opening cut out in an infinite nonuniform disk from the given temperature of the rim without a preliminary determination of the temperature field.

A method was proposed earlier which allows one to find the change in the temperature gradient at the boundary of a semiinfinite region from the given change in the temperature of the boundary without a preliminary determination of the temperature field [1, 2]. In the present report the analogous problem is solved for the steady case.

First let us consider the method in application to a well-studied problem.

The steady cylindrically symmetrical temperature field in a uniform infinite disk with a round cut-out opening, cooled from the lateral surface in accordance with Newton's law, is described by the problem

$$\left(\frac{d^2}{dr^2}+\frac{1}{r}\cdot\frac{d}{dr}-\gamma\right)T=0;\quad R\leqslant r<\infty; \tag{1}$$

$$T|_{r=R}=T_R;\ T|_{r=\infty}=0;\ \gamma=\mathrm{const}>0.$$

It is required to find the quantity $q_R = (\partial T/\partial r)_{r=R}$, which determines the heat flux to the disk.

The known solution has the form

$$T=T_R\,\frac{K_0(\sqrt{\gamma}\,r)}{K_0(\sqrt{\gamma}\,R)};\ -q_R=\sqrt{\gamma}\,\frac{K_1(\sqrt{\gamma}\,R)}{K_0(\sqrt{\gamma}\,R)}\,. \tag{2}$$

The proposed method of finding q_R without a preliminary determination of the temperature field consists in the following. We represent Eq. (1) in the form of a product of two operators, each of which contain only the first derivative with respect to r:

$$\left[\frac{d}{dr}-\sum_{n=0}^{\infty}\gamma^{\frac{1-n}{2}}b_n(r)\right]\left[\frac{d}{dr}+\sum_{n=0}^{\infty}\gamma^{\frac{1-n}{2}}a_n(r)\right]T=0. \tag{3}$$

By analogy with [1] the functions a_n and b_n can be determined using recurrent equations if one "multiplies" the operator expressions in brackets and equates terms with equal powers of $\gamma^{-1/2}$ to the original operator (1). It turns out that

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$$a_0 = b_0 = 1;\; a_1 = -b_1 = 1/2r;\; a_n = b_n,\; n \geqslant 2;$$

$$a_2 = -1/8r^2;\; a_3 = 1/8r^3;\; a_4 = -25/128r^4;\; a_5 = 13/32r^5;\; a_6 = -1073/1024r^6;\; a_7 = 103/32r^7, \ldots \tag{4}$$

Let us consider the equation formed by the right-hand cofactor of Eq. (3):

$$\left[\frac{d}{dr} + \sum_{n=0}^{\infty} \gamma^{\frac{1-n}{2}} a_n(r)\right] T = 0. \tag{5}$$

Its solution will simultaneously be solutions of the original equation (1). By analogy with [1] one can assume that all solutions of (5) automatically satisfy the condition of finiteness as $r \to \infty$. Therefore, in place of the original problem one can consider the solution of Eq. (5) with the condition $T|_{r=R} = T_R$.

The formal criterion by which we chose the right-hand cofactor in (3) rather than the left-hand consists in the following. For large enough γ Eq. (5) changes into the following equation:

$$\left(\frac{d}{dr} + \sqrt{\gamma}\right) T = 0,$$

which has a solution of the form exp $(-\sqrt{\gamma}r)$. But the solution of the left-hand cofactor has the form exp $(\sqrt{\gamma}r)$, which is incompatible with the condition of finiteness as $r \to \infty$.

By writing (5) with r = R we obtain the unknown temperature gradient in the form

$$-q_R = \sum_{n=0}^{\infty} \gamma^{\frac{1-n}{2}} a_n(R), \tag{6}$$

where $a_n(R)$ are given by the expressions (4).

By expanding the known solution (2) in a series by powers of $\gamma^{-1/2}$, using asymptotic representations of MacDonald functions, we can ascertain that Eqs. (2) and (6) are the same.

One can show that the series (6) diverges for all finite γ. Nevertheless, for large enough values of γ it is asymptotic and suitable for calculations, which results from the following argument.

Let us assume that in the series of (3) terms are calculated up to $n = N \geq 2$. By multiplying the finite series we obtain the following in place of the original equation (1):

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\cdot\frac{d}{dr} - \gamma\right) T^* + \left[\gamma^{\frac{1-N}{2}}\left(\frac{da_N}{dr} - \sum_{k=0}^{N-3} a_{N-k-1}a_{2+k}\right) - \right.$$
$$\left. -\gamma^{-N/2}\sum_{k=0}^{N-2} a_{N-k}a_{2+k} - \gamma^{-\frac{N+1}{2}}\sum_{k=0}^{N-3} a_{N-k}a_{3+k} - \ldots \gamma^{-\frac{2N-2}{2}} a_N^2\right] T^* = 0, \tag{7}$$

which is satisfied by the approximate temperature T^*. If the terms $a_1, \ldots, a_N$ are small enough, then for large γ the "contribution" to Eq. (1) is small and consequently T^* is close to the exact solution T. For example, for N = 1 we obtain in place of (7)

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\cdot\frac{d}{dr} - \gamma\right) T^* - \frac{1}{4r^2} T^* = 0,$$

from which we find

$$T^* = T_R \sqrt{\frac{R}{r}} \exp\left[\sqrt{\gamma}(R-r)\right];\quad -q_R^* = \sqrt{\gamma} + \frac{1}{2R},$$

which gives two exact terms of the expansion (2) by powers of $\gamma^{-1/2}$, with the error of the solution being determined by the expression $q_R - q_R^* \leq 1/8R^2\gamma^{1/2}$.

In application to the problem under consideration the suggested procedure gives no advantages over the known methods. However, it can be applied directly to problems with variable coefficients, and in this we see the main purpose of the present report.

We can complicate the problem discussed above by assuming that the thermal conductivity $\lambda(r, \varphi)$ of the disk depends in a known way on the coordinates:

$$\left[\lambda \frac{\partial^2}{\partial r^2} + \left(\frac{\lambda}{r} + \frac{\partial \lambda}{\partial r}\right)\frac{\partial}{\partial r} + \frac{1}{r^2}\cdot\frac{\partial \lambda}{\partial \varphi}\cdot\frac{\partial}{\partial \varphi} + \frac{\lambda}{r^2}\cdot\frac{\partial^2}{\partial \varphi^2} - \gamma\right] T = 0,$$
$$R \leqslant r < \infty, \ -\pi < \varphi \leqslant \pi, \ \gamma = \text{const} > 0, \tag{8}$$
$$T|_{r=R} = T_R(\varphi); \ T|_{r=\infty} = 0.$$

It is required to find the radial temperature gradient $q_R(\varphi) = (\partial T/\partial r)_{r=R}$ at the edge of the opening.

We represent Eq. (8) in the form

$$\left(\sqrt{\lambda}\frac{\partial}{\partial r} - \sum_{n=0}^{\infty} \gamma^{\frac{1-n}{2}} M_n\right)\left(\sqrt{\lambda}\frac{\partial}{\partial r} + \sum_{n=0}^{\infty} \gamma^{\frac{1-n}{2}} L_n\right) T = 0. \tag{9}$$

Here $M_0 = L_0 = 1$ while M_n and L_n are as yet unknown linear operators which depend, as will be seen below, on r and $\partial^k/\partial\varphi^k$. Multiplying the operators in (9), we choose expressions with the same powers of $\gamma^{-1/2}$ such that the original equation (8) is obtained. Then we obtain a system of recurrent equations for the determination of M_n and L_n:

$$\left.\begin{aligned} \frac{\partial}{\partial r}:&\ \sqrt{\lambda}(L_1 - M_1) + \frac{1}{2}\cdot\frac{\partial \lambda}{\partial r} = \frac{\lambda}{r} + \frac{\partial \lambda}{\partial r} \\ \gamma^{1/2}:&\ -L_1 - M_1 = 0 \end{aligned}\right\},$$

$$\left.\begin{aligned} \gamma^{-1/2}\frac{\partial}{\partial r}:&\ L_2 - M_2 = 0 \\ \gamma^0:&\ -L_2 - M_1L_1 - M_2 + \sqrt{\lambda}\frac{\partial L_1}{\partial r} = \frac{1}{r^2}\cdot\frac{\partial \lambda}{\partial \varphi}\cdot\frac{\partial}{\partial \varphi} + \frac{\lambda}{r^2}\cdot\frac{\partial^2}{\partial \varphi^2} \end{aligned}\right\},$$

$$\left.\begin{aligned} \gamma^{-1}\frac{\partial}{\partial r}:&\ L_3 - M_3 = 0 \\ \gamma^{-1/2}:&\ -L_3 - M_2L_1 - M_1L_2 - M_3 + \sqrt{\lambda}\frac{\partial L_2}{\partial r} = 0 \end{aligned}\right\}.$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

From this we find

$$L_1 = \frac{\sqrt{\lambda}}{2r} + \frac{1}{2\sqrt{\lambda}}\cdot\frac{\partial \lambda}{\partial r},$$
$$L_2 = -\frac{\lambda}{8r^2} + \frac{3}{8r}\cdot\frac{\partial \lambda}{\partial r} + \frac{1}{4}\cdot\frac{\partial^2 \lambda}{\partial r^2} - \frac{1}{2r^2}\cdot\frac{\partial \lambda}{\partial \varphi}\cdot\frac{\partial}{\partial \varphi} - \frac{\lambda}{2r^2}\cdot\frac{\partial^2}{\partial \varphi^2}, \tag{10}$$
$$L_3 = \frac{\sqrt{\lambda}}{2}\cdot\frac{\partial L_2}{\partial r},$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$2L_{n+1} = \sqrt{\lambda}\frac{\partial L_n}{\partial r} - \sum_{k=0}^{n-1} L_{n-1-k}L_{2+k}.$$

By writing the right-hand cofactor of Eq. (9) with r = R, as in the preceding example, we find the unknown gradient at the boundary of the region in the form of a series by powers of $\gamma^{-1/2}$:

$$-q_R(\varphi) = \sum_{n=0}^{\infty} \gamma^{\frac{1-n}{2}} L_n T_R(\varphi), \tag{11}$$

where L_n are determined from (10). For example, if $\lambda = (2 + \cos\varphi)r$ then

$$-[(2+\cos\varphi)R]^{1/2} q_R = \left\{\gamma^{1/2} + (2+\cos\varphi)^{1/2}R^{-1/2} + \frac{1}{2}\gamma^{-1/2}R^{-1}\left[\frac{1}{2}(2+\cos\varphi) + \sin\varphi\frac{\partial}{\partial\varphi} - (2+\cos\varphi)\frac{\partial^2}{\partial\varphi^2}\right] - \frac{1}{4}\gamma^{-1}R^{-3/2}(2+\cos\varphi)^{1/2}\left[\frac{1}{2}(2 + \cos\varphi) + \sin\varphi\frac{\partial}{\partial\varphi} - (2+\cos\varphi)\frac{\partial^2}{\partial\varphi^2}\right] + \ldots\right\} T_R.$$

The series in (11) is not necessarily asymptotic. For example, if $T_R = 1$, $\lambda = r^\nu$, $\nu = (4k + 2)/(2k + 3)$, and k is an integer, then $q_R = \sqrt{\gamma}K_n'(\sqrt{\gamma}R)/K_n(\sqrt{\gamma}R)$, where the index $n = (2k + 1)/2$. The MacDonald functions represent finite series by powers of $\gamma^{-1/2}$ and the ratio of polynomials is expanded in an analogous series with a finite radius of convergence.

NOTATION

T, temperature; T_R, temperature at rim of opening; T*, approximate value of temperature; q_R, radial temperature gradient at edge of opening; q_R*, approximate value of gradient; γ, heat-transfer coefficient; r, φ, polar coordinates; R, radius of opening; a_n, b_n, functions of radius; λ, thermal conductivity; M_n, L_n, linear operators; K_n, MacDonald functions.

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PHASE-TRANSITION KINETICS AND KINETIC EQUATIONS

P. M. Kolesnikov and T. A. Karpova

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Kinetic equations for gas—liquid droplet and liquid—gas bubbles systems are derived and studied by the characteristic and moment methods.

In investigating the kinetics of phase transitions in multiphase media it is necessary to study formation of phase nuclei, together with their growth and decay. The kinetics of solid-phase nucleus formation in liquid condensation and gas-phase nucleus formation in boiling are described by the well-known equations of Vollmer, Becker and Deering, Frenkel' and Zel'dovich, Courtney, Probstein, Kantrowitz, et al. [1]. Further growth of these nuclei may be considered on the basis of growth or decay kinetics of unit nuclei for monodispersed media in the absence of nucleus interaction; however, for a large number of such nuclei this approach must be replaced by a kinetic description. A number of studies have presented various kinetic equations for particle distributions over velocity and dimensions for the processes of vapor condensation, liquid or vapor crystallization [1, 2], and sublimation, boiling, and cavitation processes, but these studies usually consider distribution functions over size alone [5], or over velocity without consideration of size [6], or with consideration of size, but without distribution over velocity [1, 3].

We will present below a generalized kinetic equation for the particle distribution function f_i over time, coordinates, velocities, and particle size:

$$f_i(t,\ x,\ y,\ z,\ u,\ v,\ w,\ r). \tag{1}$$

Change in the distribution functions will be described by kinetic equations which have the form

$$\frac{\partial f_i}{\partial t}+\vec{u}_i\frac{\partial f_i}{\partial x_i}+\vec{F}_i\frac{\partial f_i}{\partial \vec{u}_i}+\frac{\partial \dot{r} f_i}{\partial r}=\sum_{i,j}J_{stij}. \tag{2}$$

For viscous liquid nuclei we may equate $\vec{F}_i$ to the Stokes friction force $\dot{u}_i = a u_i$ or to other well-known expressions, for example, $\dot{u} = a_1 u^2$, etc. Further, we assume

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